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A dynamical mechanism to explain the top-bottom quark mass hierarchy

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We discuss the mass splitting between the the top and bottom quarks in a technicolor scenario. The model proposed here contains a left-right electroweak gauge group. An extended technicolor group and mirror fermions are introduced. The top-bottom quark mass splitting turns out to be intimately connected to the breaking of the left-right gauge symmetry. Weak isospin violation occurs within the experimental limits.

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1. Introduction

In the standard model of elementary particles the fermion and gauge boson masses are generated due to the interaction of these particles with elementary Higgs scalar bosons. Despite the success of the model there are some intriguing points as, for instance, the enormous range of masses between the lightest and heaviest fermionic generations. In particular we can also quote the large ratio between the Yukawa coupling constants $Y_b/Y_t \sim 1/41$, required to give the correct masses to the top and bottom quarks. These questions could be better explained with the introduction of new fields and symmetries. One of the possibilities in this direction is the substitution of elementary Higgs bosons by composite ones in the scheme named technicolor [1,2].

The beautiful characteristics of technicolor (TC) as well as its problems were clearly listed recently by Lane [2]. Most of the technicolor problems may be related to the dynamics of the theory as described in Ref.[2]. Although technicolor is a non-Abelian gauge theory, it is not necessarily similar to QCD, and if we cannot even say that QCD is fully understood up to now, it is perfectly reasonable to realize the enormous work that is needed to abstract from the fermionic spectrum the underlying technicolor dynamics.

The many attempts to build a realistic model of dynamically generated fermion masses are reviewed in Ref.[1,2]. Most of the work in this area try to find the TC

dynamics dealing with the particle content of the theory in order to obtain a technifermion self-energy that does not lead to phenomenological problems, as an example we can consider the scheme known as walking technicolor [3]. More recent papers envisage such different dynamics as a consequence of a conformal symmetry[4]. In this work we propose a model to explain the large (t-b) splitting in the context of a technicolor scenario.

In order to obtain R $\sim \frac{1}{40}$, where R $= \frac{m_b}{m_t}$, we will consider a technicolor model based on the gauge group

$$G = SU(3)_{ETC} \otimes G' \tag{1}$$

where in the above structure we define $G' = SU(3)_{C} \otimes SU(2)_{L} \otimes SU(2)_{R} \otimes U(1)_{S}$. In this model we consider that the gauge group G' is embedded in a grand-unified theory (GUT) with a gauge group $SO(12)$ [5]. This group contains the electroweak Left-Right model that has been widely studied[6] and naturally contains a complete fermionic generation (Ψ) as well as its respective mirror partner ($\widehat{\Psi}$). An explanation of why we have performed this particular choice will become clear in the fourth section. Models along this line have been proposed by Appelquist and Schrock[7]. The variation in our case is the introduction of the mirror sector. An interesting feature of our model is that we are able of the relate the top-bottom quark mass splitting as ratio between Left-Right boson masses. The fermionic content of the model is divided into two distinct sectors, the sector (a) containing t and b quarks, T and B techniquarks and their respective leptons. The sector (b), or mirror sector ($\widehat{\ }$), contains the mirror partners $\hat{t}$ and $\hat{b}$ with respective mirror leptons.

In the above model we do not introduce an ETC gauge sector associated to the mirror fermions because the mass scale peculiar to these particles is expected to be around (1-10)TeV[8]. Therefore in this scenario it can be natural to consider that the mirror particles obtain their masses through the same mechanism responsible for the $SO(12) \rightarrow SU(3)_{C} \otimes SU(2)_{L} \otimes U(1)_{Y}$ symmetry breaking. In this model only the usual fermions and LH gauge bosons will receive their masses dynamically. The mirror fermions and the RH gauge bosons will be very heavy, and their masses could result from the existence of elementary scalar fields which can naturally appear at the GUT scale.

To ensure that $m_t \neq m_b$ without introducing large violations of weak isospin we will assume an scheme proposed by Sikivie et al. some years ago [9]. The basic idea of this mechanism is to put the chiral components (LH and RH) of a given fermion, member of an electroweak doublet, in different ETC representations. For example, in our model the chiral components of the bottom quark will be in different ETC representations, therefore the bottom quark does not receive mass at leading order as happens for the top quark. However in our model diagrams as the ones depicted in the Fig.(1) will contribute at higher order to the bottom quark mass.

2. The fermionic content of the model

According to the group structure G and following the recipe of Ref.[9] we can now choose the fermionic representation in such a way that the bottom quark mass arise from diagrams of the type shown in Fig.(1). This point will be discussed with more details in the section 4.

The fermions and mirror fermions are assigned to the following representations of the gauge group $SO(12)$ in terms of the $SO(10)$ group decomposition
(a) The fermion sector

$$\mathbf{16}^T = (t_1...t_3\,, \nu_\tau, b_1...b_3\,, \tau^-\,, b^c_1...b^c_3\,, \tau^+\,, -t^c_1... - t^c_3\,, -\nu^c_\tau)_L \tag{2}$$

(b) The mirror sector

$$\mathbf{16}^{*T} = (\hat{t}_1...\hat{t}_3\,, \hat{\nu}_\tau\,, \hat{b}_1...\hat{b}_3\,, \hat{\tau}^-\,, \hat{b}^c_1...\hat{b}^c_3\,, \hat{\tau}^+\,, -\hat{t}^c_1... - \hat{t}^c_3\,, -\hat{\nu}^c_\tau)_L \tag{3}$$

where the above structure form an irreducible spinor $\Psi^+ = (\Psi^{\mathbf{16}}\,, \widehat{\Psi}^{\mathbf{16}^*})$ of the $SO(12)$, and the index $i = 1..3$ is a color index. The $SO(N)$ gauge theories are naturally free from anomalies, the anomalies involving the ETC sector are canceled by introduction of the respective leptons and technileptons. The mirror fermions in our model are introduced in order to implement the Barr and Zee mechanism[17], which has been used to explain the muon-electron mass difference. In our case the mirror fermions will replace the exotic leptons present in the Barr and Zee model.

The fermionic assignments of the $\mathbf{16}^T$ multiplet shown in Eq.(2) with respect to the sub-group $SU(3)_{ETC} \otimes SU(3)_C \otimes SU(2)_L \otimes SU(2)_R \otimes U(1)_S$ are the following

$$\begin{array}{l}
t^i_L \sim (3,3,2,1,1/3)\ ,\ t^i_R \sim (3,3,1,2,1/3) \\
b^i_L \sim (3,3,2,1,1/3)\ ,\ b^i_R \sim (\bar{3},3,1,2,1/3) \\
\tau_L \sim (3,1,2,1,-1)\ ,\ \tau_R \sim (3,1,1,2,-1) \\
\nu_{\tau_L} \sim (3,1,2,1,-1)\ ,\ \nu_{\tau_R} \sim (\bar{3},1,1,2,-1).
\end{array}$$

The mirror fermionic assignments are similar, however mirror fermions are singlets of $SU(3)_{ETC}$, as discussed in the introduction. The technifermions quantum numbers are the same ones as the ordinary fermions. The only difference between fermions and technifermions are the TC quantum numbers. Besides that b and t are in different ETC representations.

In theories where the electroweak interaction is broken dynamically, the fermionic mass terms arise when the ETC gauge group is broken [10]. We shall not speculate about the origin of this breaking. However, we will suppose that this interaction and the $SO(12)$ gauge symmetries can be broken by fundamental scalars associated to supersymmetry or any other mechanism at the GUT scale. In the reference[5] the breaking of this GUT to $SU(3)_C \otimes SU(2)_L \otimes U(1)_Y$ is discussed in detail. For convenience we will reproduce the symmetry breaking pattern of this

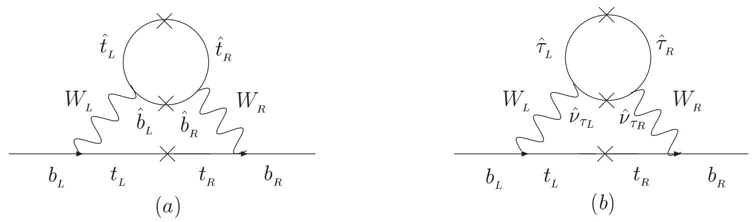

Fig. 1. Diagrams contributing to the bottom quark mass. In the diagram (a) the contribution to the b quark mass comes from the mirror-quarks of our model while the contribution of the diagram (b) is due to the mirror-leptons.

model

$$\begin{aligned} SO(12) &\xrightarrow{M_a} SU(4)_F \otimes SU(4)_C \\ &\xrightarrow{M_b} SU(4)_C \otimes SU(2)_L \otimes SU(2)_R \otimes U(1)_F \\ &\xrightarrow{M_c} SU(3)_C \otimes SU(2)_L \otimes SU(2)_R \otimes U(1)_{C+F} \\ &\xrightarrow{M_d} SU(3)_C \otimes SU(2)_L \otimes SU(2)_R \otimes U(1)_S \\ &\xrightarrow{M_R} SU(3)_C \otimes SU(2)_L \otimes U(1)_Y . \end{aligned} \quad (4)$$

where we indicate $U(1)_{C+F} = U(1)_C \otimes U(1)_F$. The masses scales $(M_{a..d}, M_R)$ related to the symmetry breaking of $SO(12)$ group down to $SU(3)_C \otimes SU(2)_L \otimes U(1)_Y$ are determined respecting the constraints on the proton decay lifetime and the renormalization of the weak angle bare value $\sin^2\theta_W$ to the experimental value $\sin^2\theta_W = 0.23120(15)$[11]. In our model the symmetry breaking of the last stage shown in Eq.(4) will be implemented by a techniquark condensate characterized by a strong interaction $SU(2)_{TC}$ symmetry at $O(300)GeV$. According to what we discussed above we assume that the breaking of the $SU(3)_{ETC} \to SU(2)_{TC}$ will happens near the GUT scale.

We shall concentrate only in the low energy sector (i.e. bellow 1 TeV) of our model, $SU(3)_C \otimes SU(2)_L \otimes SU(2)_{TC} \otimes U(1)_Y$, and in particular in the top-bottom mass splitting without touching the problem of mass generation for the remaining fermions.

3. The Top Dynamical Mass

We can decompose the fermionic content associated to the gauge group $SU(3)_{ETC}$ as shown in the following:

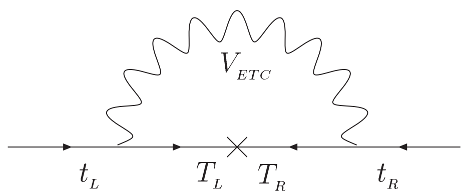

Fig. 2. Diagram responsible for the top quark mass.

$$\psi_{\mathbf{3}_{ETC}} = \begin{pmatrix} T_{1,\omega} \\ T_{2,\omega} \\ t \end{pmatrix}_{L,R} , \quad \begin{pmatrix} B_{1,\omega} \\ B_{2,\omega} \\ b \end{pmatrix}_{L} \sim \quad (\mathbf{3}, 1)$$
$$\psi_{\bar{\mathbf{3}}_{ETC}} = \begin{pmatrix} B_{1,\omega} \\ B_{2,\omega} \\ b \end{pmatrix}_{R} \sim \quad (\bar{\mathbf{3}}, 1) \qquad (5)$$

where ω is a techniflavor index indicating the number(n_ω) of flavors necessary to produce the walking behavior for the TC group. In Eq.(5) we indicate only the $SU(3)_{ETC}$ decomposition, the $SU(2)_L$ structure of $(t\,,\,b)_L$ multiplet is the usual one.

This choice is in agreement with the scheme proposed by Sikivie et al. [9] in order to assure that $m_b \neq m_t$ without large weak isospin violation, where we are putting the (LH - RH) chiral components of the top quark in the same ETC representation, but the bottom quark components are in different ETC representations. In this way the dynamical top quark mass is generated at leading order as depicted in the Fig.(2), as happens in ordinary technicolor models.

The top quark mass is giving by the following expression:

$$m_t \approx \frac{3g^2_{ETC}C_{ETC}}{8\pi^2}\int^{\Lambda_{ETC}} \frac{d^4p}{(p^2+\Lambda^2_{TC})}\frac{\Sigma(p)_{TC}}{(p^2+\Lambda^2_{ETC})}, \qquad (6)$$

while at this order we have $m_b = 0$, and Λ_{ETC} is the mass scale of the ETC bosons. It is useful to write the technifermion self-energy in terms of the ansatz formulated in the Ref. [12]

$$\frac{\Sigma(p)_{TC}}{\Lambda_{TC}} = \left(\frac{\Lambda^2_{TC}}{p^2}\right)^{\alpha}[1+bg^2(\Lambda^2_{TC})ln(p^2/\Lambda^2_{TC})]^{-\gamma_{TC}cos(\alpha\pi)} \qquad (7)$$

where, in this equation, we identify $\gamma_{TC} = \frac{3C_{TC}}{16\pi^2 b}$ with C_{TC} given by $C_{TC} = \frac{1}{2}[C_2(R_1) + C_2(R_2) - C_2(R_3)]$, where $C_2(R_i)$ (with $i = 1..3$) is the Casimir operator for the technifermions in the representations R_1 and R_2 that condensate in the representation R_3, and b is the coefficient of order g^3 in the technicolor β_{TC} function. The quadratic Casimir operator for the ETC gauge group can be written as $C_{ETC} = (N^2_{ETC} - 1)/2N_{ETC} = 4/3$ for $N_{ETC} = 3$. Finally, the mass

scale Λ_{TC} can be estimated to be of the order of the electroweak symmetry scale $\Lambda_{TC} \sim O(300)\, GeV$, according the discussion performed in the last section the scale Λ_{ETC} can be assumed as $\Lambda_{ETC} \approx \Lambda_{GUT}$.

Eq.(7) interpolates between the standard operator product expansion result for the technifermion self-energy, which is obtained when $\alpha \to 1$, and the extreme walking technicolor solution obtained when $\alpha \to 0$ [3]. As we have pointed out in Ref. [13] only such kind of solution is naturally capable of generating a large mass to the third fermionic generation, which has a mass limit almost saturated by the top quark mass. Moreover, as also claimed in the second paper of Ref.[13] there are other possible reasons to have $\alpha \sim 0$, as the existence of an IR fixed point and a gluon (or technigluon) mass scale [15], which are closely related [16].

We will just consider $\alpha = 0$ in Eq.(7) assuming that we can introduce a number of technifermions necessary to obtain the extreme walking behavior, our result for the t quark mass (and consequently for b quark mass) will be dicussed exactly in this case. Therefore, after its substitution into Eq.(6), with $p^2 = (\Lambda^2_{ETC}/\Lambda^2_{TC})y$, we verify that Eq.(6) is reduced to

$$m_t \approx \frac{3g^2_{ETC} C_{ETC} \Lambda_{TC}}{16\pi^2} \left[1 + bg^2(\Lambda^2_{TC}) ln(\frac{\Lambda^2_{ETC}}{\Lambda^2_{TC}}) \right]^{-\gamma_{TC}} I(\Lambda^2_{TC}) \tag{8}$$

where we used the definition

$$I(\Lambda^2_{TC}) = \frac{1}{\Gamma(\delta)} \int_0^\infty dz e^{-z} z^{\delta-1} \left(\frac{1}{\Lambda^2_{TC}} \right)^{-\epsilon z} \int^{\Lambda^2_{TC}} \frac{dy y^{-\epsilon z}}{y + \Lambda^2_{TC}}. \tag{9}$$

To obtain the above equation we made use of the following Mellin transform:

$$\left[1 + \epsilon \ln \frac{A}{B} \right]^{-\delta} = \frac{1}{\Gamma(\delta)} \int_0^\infty dz z^{\delta-1} e^{-z} \left(\frac{A}{B} \right)^{-\epsilon z} \tag{10}$$

and defined $\epsilon = bg^2(\Lambda^2_{TC})$, with $\delta = \gamma_{TC} + 1$. After integration of the Eq.(9) we can show that Eq.(8) leads to [13]

$$m_t \sim C_{ETC} \alpha_{ETC} \Lambda_{TC}. \tag{11}$$

The above result, which can naturally describe the top quark mass [13], resembles the way the top quark acquires a mass from an elementary Higgs boson with vacuum expectation value Λ_{TC}. This behavior could be predicted based on the results of Ref.[14], where it is remarked that when $\alpha \sim 0$ in Eq.(7) the composite Higgs system behaves as a fundamental one, and in this case some results obtained with the introduction of fundamental Higgs bosons are reproduced in a dynamical scenario. This fact underlies several aspects of our model.

4. The bottom mass

According to what we discussed at the beginning of this work, the bottom quark mass will be generated at 2-loop level, due to the interaction with mirror-quarks(leptons) and the $W_{L,R}$ bosons as depicted in Fig.(1). There are other possible contributions to the b quark mass, but they involve the very massive $SO(12)$

gauge bosons and are negligible compared to the ones of Fig(1). Assuming the usual Feynman rules, we can write the following expression for the b mass

$$m_b = -i\frac{\alpha_L \alpha_R}{\pi^2}\int \frac{d^4q}{(q^2-m_t^2)}\left[\Gamma^{\mu} D_{\mu\beta}(p-q) I^{\beta\delta}(l,p,q) D_{\delta\nu}(p-q)(\not{q}+m_t)\Gamma^{\nu}\right] \tag{12}$$

where in the above equation we defined the function $I^{\beta\alpha}$ as the fermion loop integral

$$I^{\beta\delta}(l,p,q) = i\int \frac{d^4l}{(2\pi)^4} Tr\left[\Gamma^{\beta} S_F(l+p-q)\Gamma^{\delta} S_F(l)\right]. \tag{13}$$

In these expressions we are introducing the following notation: $\Gamma^{\mu..\nu} = \gamma^{\mu..\nu}\sigma^{a..d}$. The $\sigma^{a..d}$, for $a..d = 1,2$ or 3, are $SU(2)_{L,R}$ generators and the $D_{\mu\nu}(p-q)$ is the $W_{L,R}$ propagator in the Landau gauge. Eq.(13), after angular and l integration, can be written as

$$\frac{m_b}{m_t} \approx -f(\alpha, n_{\hat{F}})\int \frac{dq^2}{(q^2+m_t^2)}\frac{(q^2+M_{\hat{t}}^2)}{(q^2+M_L^2)}\frac{q^2}{(q^2+M_R^2)}. \tag{14}$$

where $M_{\hat{t}}^2 = 3m_{\hat{t}}^2/4$. In order to obtain this last equation, for simplicity, we considered the zero momentum approximation for the external propagators, we neglected the momentum dependence of the fermion self-energies inside the loop and assumed only the first diagram shown in the Fig.(1). The masses m_t and $m_{\hat{t}}$ are respectively the top and mirror-top masses, M_L and M_R are the weak vector boson masses and we introduced the function

$$f(\alpha, n_{\hat{F}}) = \frac{27}{64\pi^2}\alpha_L \alpha_R n_{\hat{F}} C_{2_F}^L C_{2_F}^R, \tag{15}$$

with $n_{\hat{F}}$ indicating the number of mirrorquarks(leptons) and $C_{2_F}^L = C_{2_F}^R = 3/4$. Notice that if we had considered the momentum dependence in the self-energies of Fig.(1) we would be lost in factors that do not bring any useful information to our calculation. Moreover, as we argue in the sequence, even in this approximation the calculation is finite.

Simple power counting shows that the integral in Eq.(14) is *apparently* logarithmically divergent, we stress two points: First, if we had considered the momentum dependence in the self-energies of Fig.(1) the integral would be softened. Second, and as well as important, the model that we are proposing here bears some similarity with the one proposed by Barr and Zee some years ago [17], whose purpose was the calculation of the electron mass in terms of the muon mass in the case of elementary scalar bosons. In that model a “similar” type of logarithmic divergence also occurs, which was canceled among different contributions to the electron mass; the muon contribution and the contribution of a heavy lepton X. This cancellation occurs when it is assumed that the muon mixes with this lepton X. In our model we can suppose that the top and mirror top mixes at the $SO(12)$ scale, and the

cancellation now will occur between the fermion and mirror fermion sector keeping the b mass finite. In terms of this mixing we can write Eq.(14) in the following form

$$\begin{aligned} m_b \approx & -m_t(\omega) f(\alpha, n_{\hat{F}}) \int \frac{dq^2}{(q^2+m_t^2)} \frac{(q^2+M_{\hat{t}}^2)}{(q^2+M_L^2)} \frac{q^2}{(q^2+M_R^2)} + \\ & -m_{\hat{t}}(\omega) f(\alpha, n_{\hat{F}}) \int \frac{dq^2}{(q^2+m_{\hat{t}}^2)} \frac{(q^2+M_{\hat{t}}^2)}{(q^2+M_L^2)} \frac{q^2}{(q^2+M_R^2)} . \end{aligned} \tag{16}$$

where, for simplicity, we defined

$$\begin{aligned} m_t(\omega) &= m_t(\cos\omega_L)(\cos\omega_R) \\ m_{\hat{t}}(\omega) &= m_{\hat{t}}(\sin\omega_L)(\sin\omega_R). \end{aligned} \tag{17}$$

The relation between m_t , $m_{\hat{t}}$ and the mixing angles $\omega_{L,R}$ as in the case of the Barr and Zee model[17] can be parametrized as $m_t(\cos\omega_L)(\cos\omega_R) = -m_{\hat{t}}(\sin\omega_L)(\sin\omega_R)$. Then, after the integration of the Eq.(16), we can write the divergent pieces of this equation as

$$m_b(\Lambda) \propto m_t(\cos\omega_L)(\cos\omega_R)\log\Lambda^2 - m_t(\cos\omega_L)(\cos\omega_R)\log\Lambda^2. \tag{18}$$

Now we can assume that $\Lambda \to \infty$ because the combination of the two terms shown above give the finite result $m_b(\Lambda) = 0$.

In the model of reference[18], which has also a common structure to the model discussed here, the authors estimated the mixing angle of the ordinary fermions with their mirror partners as being $\omega \leq 10^{-3}$, meaning that its cosine can be approximated by 1. Therefore our model has a larger convergence than the one in the Barr and Zee model due to the points discussed above. Now we can justify our choice for $SO(12)$ in order to unify the interactions at the scale M_{GUT}, because this gauge group exactly accommodates one usual fermionic generation and one mirror fermion generation necessary to keep the bottom mass finite in the present approach. This happens independently of the fact that we neglected the momentum dependence of the self-energies in the loops of Fig.(1).

Finally, we can evaluate the contributions to the bottom quark mass shown in Fig.(1). Diagram (b) in Fig.(1) gives a contribution identical to the diagram (a), and the second can be obtained from the first one just exchanging in the calculation the mass $m_{\hat{t}}$ by the mirror-lepton masses $m_{\hat{\tau}}$. Taking into account both contributions we can write the ratio between the bottom and top quark masses as

$$\frac{m_b}{m_t} \approx \frac{243}{1024\pi^2}\alpha_L\alpha_R n_{\hat{F}} \frac{M_R^2}{(M_R^2 - M_L^2)} \left[\frac{(M_{\hat{t}}^2 - M_R^2)}{(m_t^2 - M_R^2)} + \frac{(M_{\hat{\tau}}^2 - M_R^2)}{(m_t^2 - M_R^2)} \right] \log\frac{M_R^2}{M_L^2} \tag{19}$$

where we have already set $\cos\omega_{L,R} \approx 1$.

It is important to note that in this model the mass scale $M_{R(L)}$ will be linked with the symmetry breaking scale of the $SO(12)$ group by the following relations[5]

$$1 - \frac{8}{3}\sin^2\theta_W = \frac{11\alpha}{6\pi}\left[\frac{4}{3}\log\frac{M_{GUT}^2}{M_L^2} + 2\log\frac{M_R^2}{M_L^2}\right] \tag{20}$$

and

$$\frac{\alpha}{\alpha_s} - \sin^2\theta_{_W} = \frac{11\alpha}{6\pi}\left[\log\frac{M^2_{_R}}{M^2_{_{GUT}}} - \log\frac{M^2_{_R}}{M^2_{_L}}\right]. \tag{21}$$

In order to keep $\sin^2\theta_{_W} = 0.231$ and $M^2_{_{GUT}} \propto 10^{16}GeV$ the symmetry breaking connected to the scale $M_{_R}$ must happens at $M_{_R} \sim 10^{13}GeV$. Therefore, we can disregard the fermionic(mirror-fermionic) masses in comparison with the $M_{_R}$ mass and put Eq.(19) into the following form

$$\frac{m_b}{m_t} \approx \frac{243}{512\pi^2}\alpha_{_L}\alpha_{_R}n_{_{\hat{F}}}\frac{M^2_{_R}}{(M^2_{_R} - M^2_{_L})}\log\frac{M^2_{_R}}{M^2_{_L}}. \tag{22}$$

This last equation is independent of the mirror fermions masses introduced in the model and as the Right-Handed symmetry is broken near at GUT scale the contribution associated to this very high energy scale does not contribute to the ρ parameter. We can regard this contribution as $\delta\rho \propto M^2_{_L}/M^2_{_R}$, therefore we do not have too large corrections to the Peskin-Takeuchi T parameter, which is giving by 19

$$\alpha T = \frac{M^2_W}{M^2_Z\cos^2\theta_W} - 1 = \rho - 1 = \delta\rho. \tag{23}$$

In the figure below we plot the behavior of the bottom quark mass as a function of the ratio between the $M_{_R}$ and $M_{_L}$ masses.

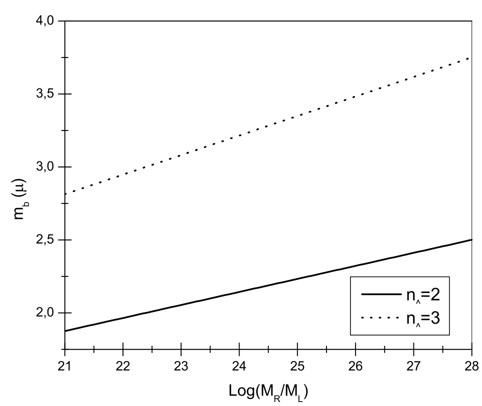

Fig. 3. The behavior of the bottom quark mass, $m_b(\mu)$, as a function of $\frac{M^2_R}{M^2_L}$. In the box we are indicating the number of mirror flavors $n_{\hat{}} \equiv n_{_{\hat{F}}} = 2$ and $n_{_{\hat{F}}} = 3$ that we consider in our model. Only the strong radiative corrections have been considered to obtain these curves.

The values of Fig.(2) correspond to the bottom quark mass at the scale $\mu^2 = 10\,GeV^2$, assuming that the bottom quark receives strong radiative corrections at low energies according to the correction factor given by [20]

$$m_b(\mu) \approx m_b(\Lambda_{GUT}) \left(\frac{\alpha_{QCD}}{\alpha_{GUT}} \right)^{4/(11-\frac{2}{3}n_f)} . \tag{24}$$

For simplicity we considered only the QCD radiative corrections and we also assumed $\alpha_L = \sqrt{2} G_F M_L^2/\pi \sim 1/21$, $\alpha_R \sim 1/37$[a], $\alpha_{GUT} \sim 1/45$, $\alpha_{QCD}(\mu^2 = 10GeV^2) \approx 0.32$ with $m_t(\mu \approx 2M_z) \approx 178\,GeV$ and $n_f = 6$. G_F and α_{QCD} are respectively the Fermi coupling constant and QCD coupling constant.

5. Conclusions

In this model we were able to generate a reasonable mass splitting between the bottom and top quarks without introducing large weak isospin violation. This is possible because we followed the Sikivie et al. [9] scheme, according to which the technicolor interaction preserves a custodial $SU(2)_V$ symmetry.

The ratio $R \sim \frac{1}{40}$ between the bottom and top quark masses that arises in our model is intimately connected to the breaking of the Right-Handed symmetry. Of course, we are trading the problem of understanding the large mass splitting between the top and bottom quarks, to the more fundamental question, of understanding why there is a very large splitting between the Left-Right gauge symmetries. There are several points in the model that still need to be developed as the introduction of the other families, the origin of the ETC symmetry breaking as well as of the GUT symmetry, etc... However we expect that the basic idea of the model discussed here may represent a different approach to solve this problem.

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